APPENDIX B

Matrices, Vectors and Calculus

Contents

B.1 Introduction

In this appendix B, I want to summarize key results on matrix operations and calculus that are needed in principal component analysis. Readers wanting to understand how the principal components are derived should be comfortable with some basic rules that govern matrix operations and matrix calculus. Moreover, results on the differentiation of scalar functions with respect to a vector or a matrix will be presented. Also presented in this appendix are results and properties related to the differentiation of vector functions with respect to a scalar or a vector.

B.2 Rules of Matrix Operations

In the linear algebra literature, a vector is typically considered to be a column vector. That is, a vector $\boldsymbol{x}$ with 3 elements x_1, x_2 and x_3 for example is defined as,

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

You can also represent this vector in a more compact format as $\boldsymbol{x} = (x_1, x_2, x_3)^\top$ or $\boldsymbol{x} = (x_1 \quad x_2 \quad x_3)^\top$, where the symbol $\top$ indicates that the entity it applies to must be transposed vertically.

B.2.1 *Scalar product of 2 vectors*

Consider 2 vectors of the same size $\boldsymbol{x}$ and $\boldsymbol{y}$ given by,

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \text{ and } \boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}.$$

The scalar product of $\boldsymbol{x}$ and $\boldsymbol{y}$ is a scalar obtained by summing all elementwise products. That is,

$$\boldsymbol{x}^\top \boldsymbol{y} = \boldsymbol{y}^\top \boldsymbol{x} = x_1 y_1 + x_2 y_2 + x_3 y_3.$$